COVID-19 Pandemic Volterra Integral Equation Model

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Introduction

The COVID-19 pandemic has affected many people throughout not just the U.S. but across the whole world. The objective of this research project is to find a numerical solution through the Galerkin Method for the Volterra Integral Equation Model. The non-homogenous Volterra Integral Equation of the second kind is used to capture a broader range of disease distributions. Volterra Integral equation models are used in the context of mathematics, public health, and evolutionary biology. The mathematical model of this integral equation will yield convergence results for the COVID-19 data for Italy. The modeling of this country will be done using the Galerkin method, a type of numerical approximation using the Gaussian Quadrature nodes. Inspired by the COVID-19 pandemic, the model will include the number of initially infected individuals, the rate of infection, contact rate, death rate, fraction of recovered individuals, and the mean time an individual remains infected.

Volterra Integral

The nonhomogeneous Volterra integral equation of the second kind was used, where K(t,s) is the kernel of the integral equation, and λ is a parameter.:

$$y(t) = f(t) + \lambda \int_a^t K(t,s)y(s)ds$$

Galerkin and Gaussian Quadrature Methods

We approximate the integrals by a sum by using the Galerkin Method, as the area of approximation is complex. In order to use the Galerkin Method, we first convert the original integral to an integral bounded from -1 to 1. Since the integrals in the IRCD Model do not have a closed form we use the Gaussian Quadrature method where n=5 to approximate the integrals. The Gaussian Quadrature method works as follows:

$$\int_a^b f(x)\,dx = \int_{-1}^1 f\left(\frac{(b-a)t + (b-a)}{2}\right)\left(\frac{b-a}{2}\right)dt$$

Then the Galerkin Method is applied:

$$\int_a^b f(x)\,dx = \sum_{i=1}^n c_i p(x_i)$$

where $p(t) = f\left(\frac{(b-a)t+(b-a)}{2}\right)\left(\frac{b-a}{2}\right)$, and c_i and x_i are given as follows for n=5:

$c_1=c_5=0.236926885$, $c_2=c_4=0.4786286705$, and $c_3=0.56888888889$

$x_1=-x_5=0.9061798459$, $x_2=-x_4=0.5384693101$, and $x_3=0$

IRCD Model

The Infected, Recovered, Contact, Death (IRCD) Model is given below:

$$I(t) = I_o + \frac{R\lambda(t)bP(t)}{\lambda_o\tau}\int_0^t I(s)\big(1 - I(s)\big)ds$$

$$I(t) = Fraction\ of\ infected$$
$$t = Days\ since\ February\ 15, 2020$$
$$I_o = Fraction\ that\ are\ initially\ infected\ (at\ t = 0)$$
$$R = Average\ rate\ of\ infection$$
$$\lambda(t) = Rate\ of\ contact$$
$$b = Average\ rate\ of\ deaths$$
$$P(t) = Fraction\ that\ are\ recovered$$
$$\lambda_o = Initial\ contact\ rate = 12t$$
$$\tau = Mean\ time\ the\ individual\ remains\ infected = \int_0^t P(s)e^{-bs}ds$$

Logistic 4P Fit

For Italy, the JMP logistic 4P fit for all three models (total cases, total deaths, and number recovered) uses the following equation:

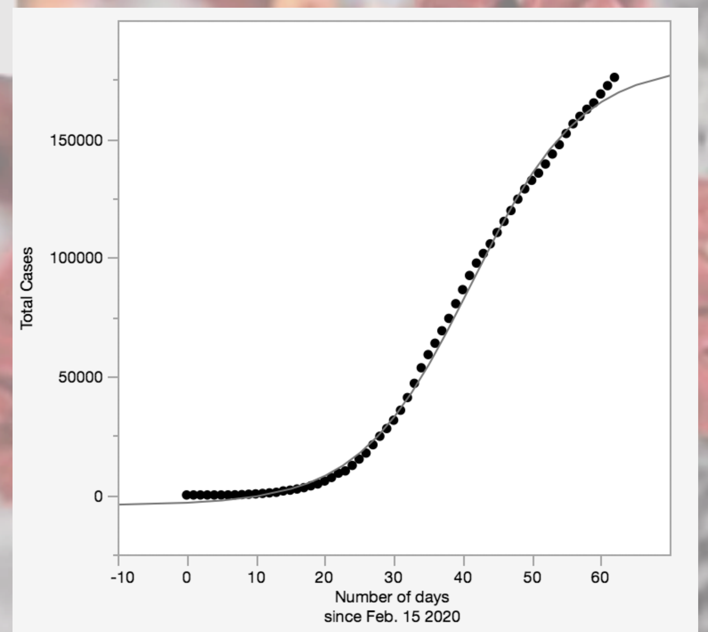
$$c + \frac{d-c}{1+e^{-a(t-b)}}$$

where,

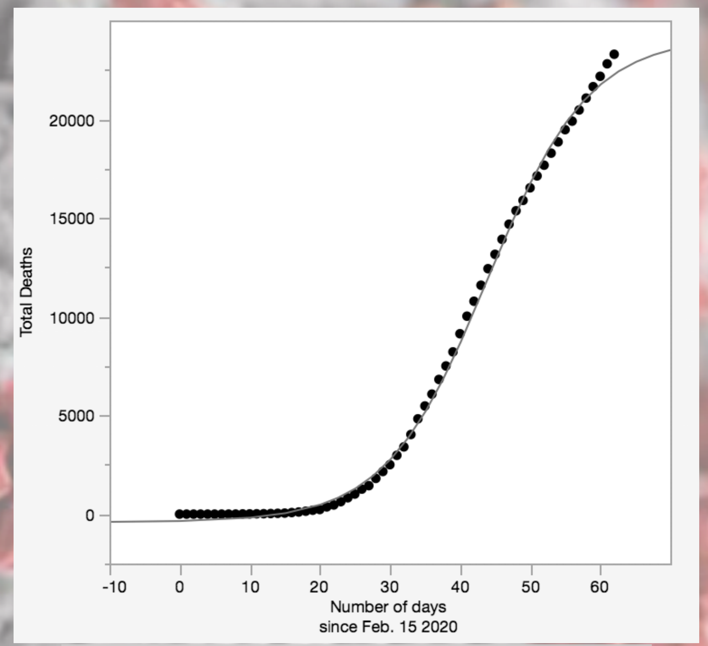
$$a = Growth\ Rate, b = Inflection\ Point, c = Lower\ Asymptote, and$$
$$d = Upper\ Asymptote$$

	Total Cases	Total Deaths	Number Recovered
Growth Rate (a)	0.1246267	0.1377106	0.1239545
Inflection Point (b)	41.073556	43.80613	40.564131
Lower Asymptote (c)	-3993.021	-362.7532	-3645.437
Upper Asymptote (d)	182033.86	24205.583	157343.37

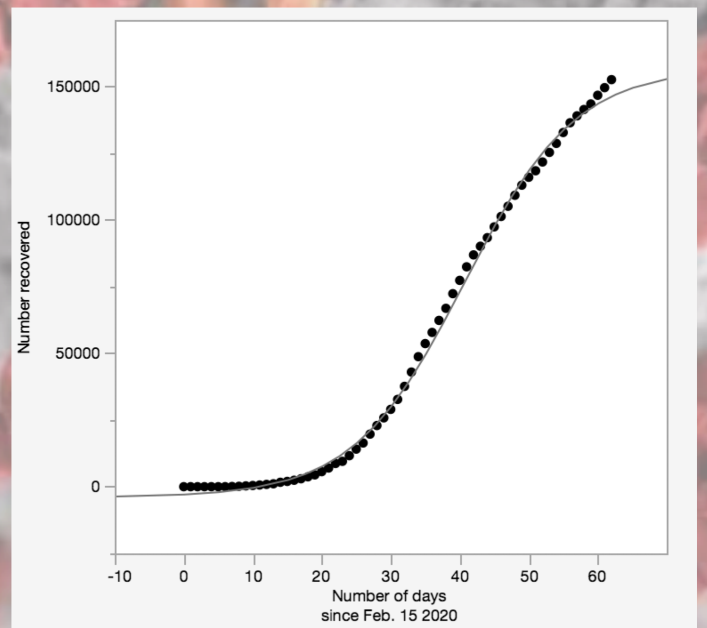
This table shows the values of a, b, c, and d for the three Logistic 4P fits for total cases, total deaths, and number recovered.



The logistic 4P fit for the total cases vs. the number of days since February 15, 2020



The logistic 4P fit for the total deaths vs. the number of days since February 15, 2020



The logistic 4P fit for the umber of recovered vs. the number of days since February 15, 2020

Assumptions and Drawbacks

- The assumption was made that the contact rate between people during this pandemic decreased to the average family size multiplied by the number of days.
- The initial data was gathered starting at February 15, 2020.
- Gaussian Quadrature n=5 was used to approximate the integrals.
- Since a small number of nodes for the Gaussian Quadrature method were used, the model is only 90% accurate from 50 to 100 days after February 15, 2020.

Italy

A logistic 4P fit was used to fit a curve to the number of infected, number of deaths, and number of recovered graphs for Italy. By using a logistic 4P fit, the three fitted curves each had an R-Squared value of 0.998, and Chi-Squared values less than 0.0001. From these three graphs, the six variables were calculated and found. The Number of Cases vs Time plot was used to find I_0 , I(s), and R, where R is just the average of the derivative of I(s). The Number of Deaths vs Time plot was used to find the variable b by taking the average of the derivatives from the given fit. Lastly, the Number of Recovered vs Time plot was used to find P(t) which is the equation for the line of fit. τ was found afterwards using P(t) and b that were already found.

$$I(t) = What\ we\ are\ solving\ for$$
$$t = The\ amount\ of\ days\ after\ February\ 15, 2020\ that\ we\ want\ to\ find\ I(t)\ at$$
$$I_0 = 3$$
$$I(s) = -3993 + \frac{186027}{1+e^{-0.125t+5.12}}$$
$$R = 950.52$$
$$\lambda(t) = 2.58t$$
$$b = -14t$$
$$P(t) = -3645 + \frac{160988.8}{1+e^{-0.12t+5.03}}$$

Number of Days	Actual Total Number of Infected People	IRCD Predicted Total Number of Infected People	Error
50	132,547	127,874	$3.526*10^{-2}$
75	207,415	190,826	$7.998*10^{-2}$
100	230,553	252,593	$9.559*10^{-2}$

This table shows the actual number of infected people from the COVID-19 data for the given days (50, 75, and 100 days) and the number of infected people predicted by the IRCD model. The error from the model is also shown.

Conclusion

- The IRCD Model approximated using the Gaussian Quadrature method is 90% accurate from 50 to 100 days after February 15, 2020.
- This model has the potential to accurately model and predict the future of COVID-19 cases in Italy and countries with similar COVID-19 pandemic patterns where we plan to approximate the integrals with a higher number of Gaussian Quadrature nodes by using FORTRAN-77.

Future Work

- In the future, FORTRAN-77 will be used to approximate the integral with a higher number of Gaussian Quadrature nodes. By doing so the IRCD Model should be more accurate for any time, t.
- The model will be extended to the whole country of the United States after the COVID-19 infection slows down.
- The function modeling the death rate will be modified from a linear function to an exponential function after increasing the Gaussian Quadrature nodes for a more realistic picture of the model.

References

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