Simulations of Transcranial Electrical Stimulation with Variable Tissue Conductivities

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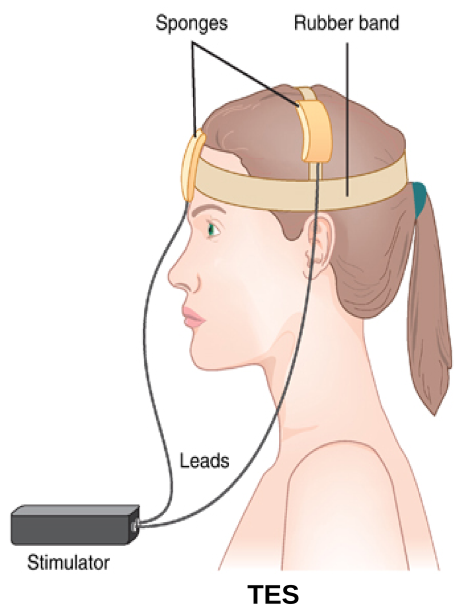
Background

Transcranial Electrical Stimulation (TES)

- Noninvasive therapy that applies low doses of electrical current directly to a patient's head surface
- Utilizes electrodes positioned on scalp with the goal of enhancing neuronal functioning
- Shown to be effective in mitigating symptoms of neurodegenerative diseases such as Parkinson's disease and Alzheimer's disease [1]



Motivation

- Current TES simulations simply a standard conductivity value for the tissues of the head cavity
- However, these conductivity values can vary within the tissue and between patients
- Incorporating variability may show to be important for TES simulations to accurately predicting electrical current delivery

Research Goal

- Incorporate tissue conductivity stochasticity into TES computational simulations
- Assess the impact of variability in electrical conductivity on patient-specific TES simulation results

Model and Simulations

Mathematical Model

$$\nabla \cdot \mathrm{M}\nabla\Phi = 0, \quad \mathbf{x} \in \Omega$$

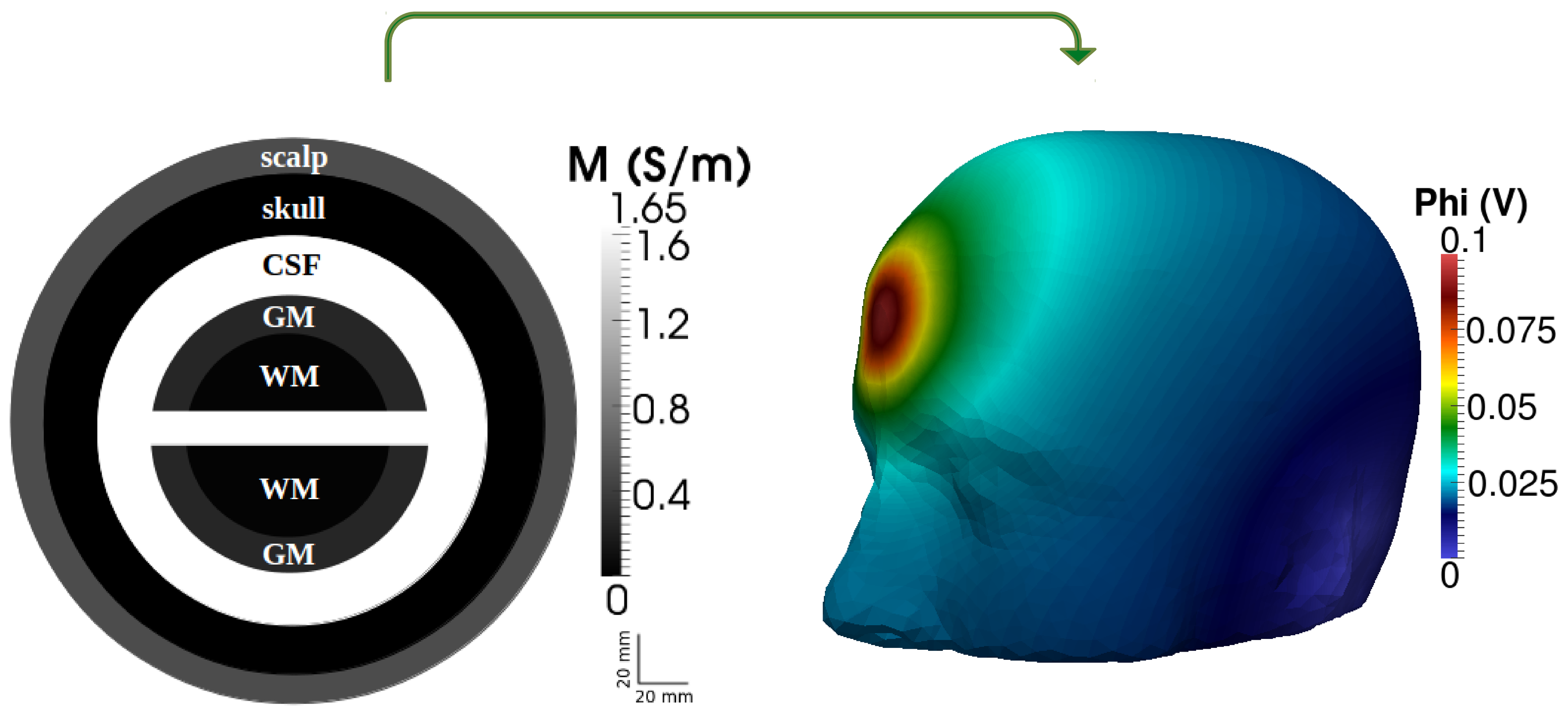
$$\Phi = 0, \quad \mathbf{x} \in \partial\Omega_C$$

$$\mathbf{n} \cdot \mathrm{M}\nabla\Phi = I(\mathbf{x}), \quad \mathbf{x} \in \partial\Omega_A$$

$$\mathbf{n} \cdot \mathrm{M}\nabla\Phi = 0, \quad \mathbf{x} \in \partial\Omega_S$$

- Head and brain is viewed as a passive volume conductor
- Models electric potential and electric current
- Boundary conditions for anode (+), cathode (ground), and remainder of scalp

Simulation Domain



Implementation

- **Finite Element Method:** Numerical Method to Solve PDE
- **Weak Formulation:**

Find $\Phi \in H_0^1(\Omega)$ such that

$$\int_\Omega \nabla v \cdot \mathbf{M}\nabla\Phi \, dx = \int_{\partial\Omega_A} vI \, ds \quad \forall\, v \in H_0^1(\Omega),$$

where

$$H_0^1(\Omega) = \{u \mid u \in H^1(\Omega), u = 0 \;\forall\, \vec{x} \in \partial\Omega_C\},$$

$$H^1(\Omega) = \{u \mid u \in L_2(\Omega), \frac{\partial u}{\partial x_i} \in L_2(\Omega)\},\; i = 1, ..., d,$$

and

$$L_2(\Omega) = \{p \mid \int_\Omega |p|^2 dx < \infty\}.$$

Computational Tools

- **FEniCS** (Python)- Used for computing partial differential equations using the finite element method for circle results
- **Gmsh**- Used to create the computational domains
- **Paraview**- Used for visualization

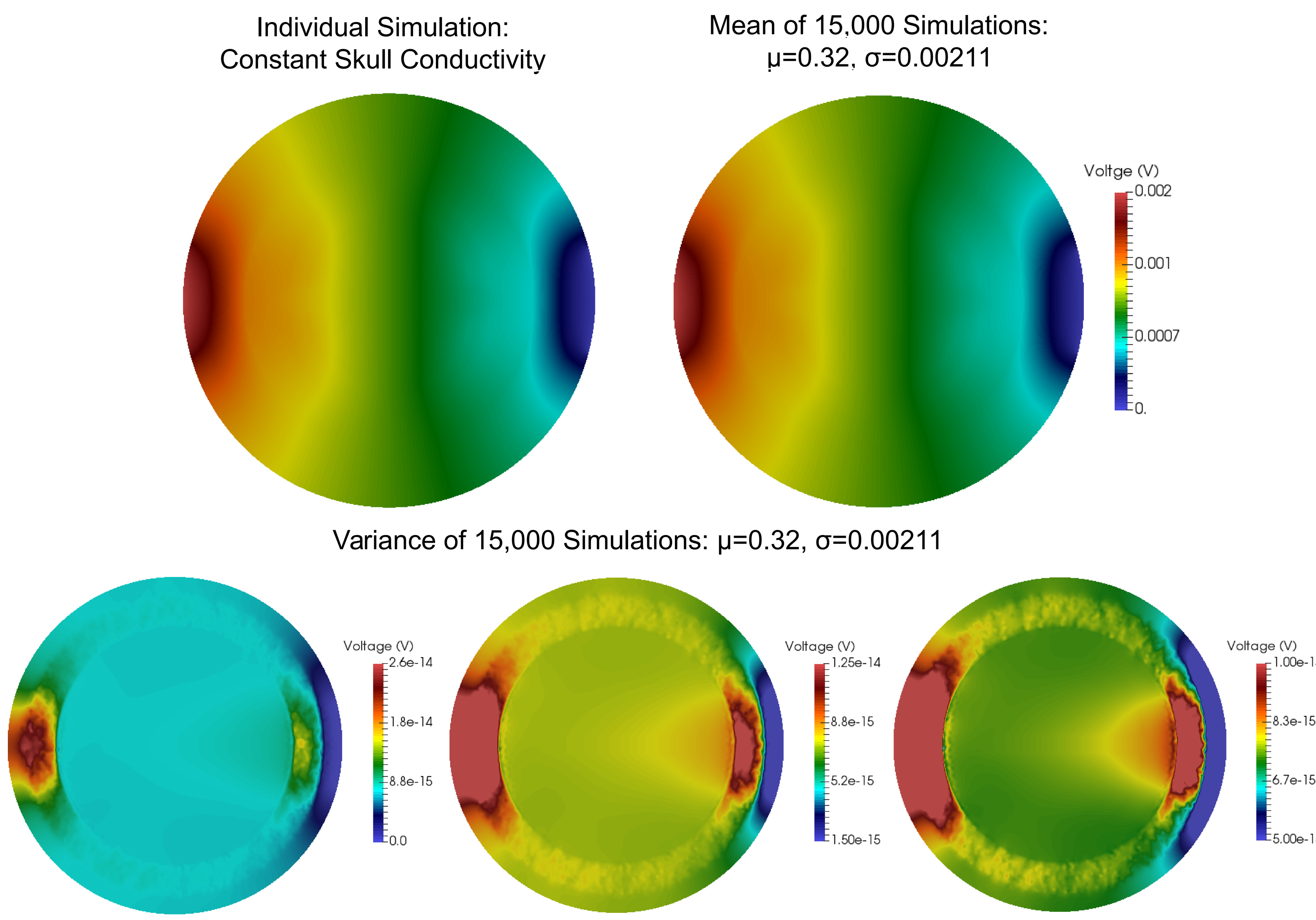
Progress and Results

Current Results

- Biologically-based variability in skull tissue conductivity impacts TES simulation prediction; the depths of current density into the head cavity are notably affected
- Preliminary results suggest further differences in current density depths due to variability in conductivities of the other cranial tissues
- The skull is known to be a barrier tissue of TES due to its extremely low conductivity that shunts TES energy, thereby effectively shielding brain matter from the TES current density; accurately simulating skull conductivity variability is therefore essential to properly predict current density target locations and saturation depths in computational simulations of TES

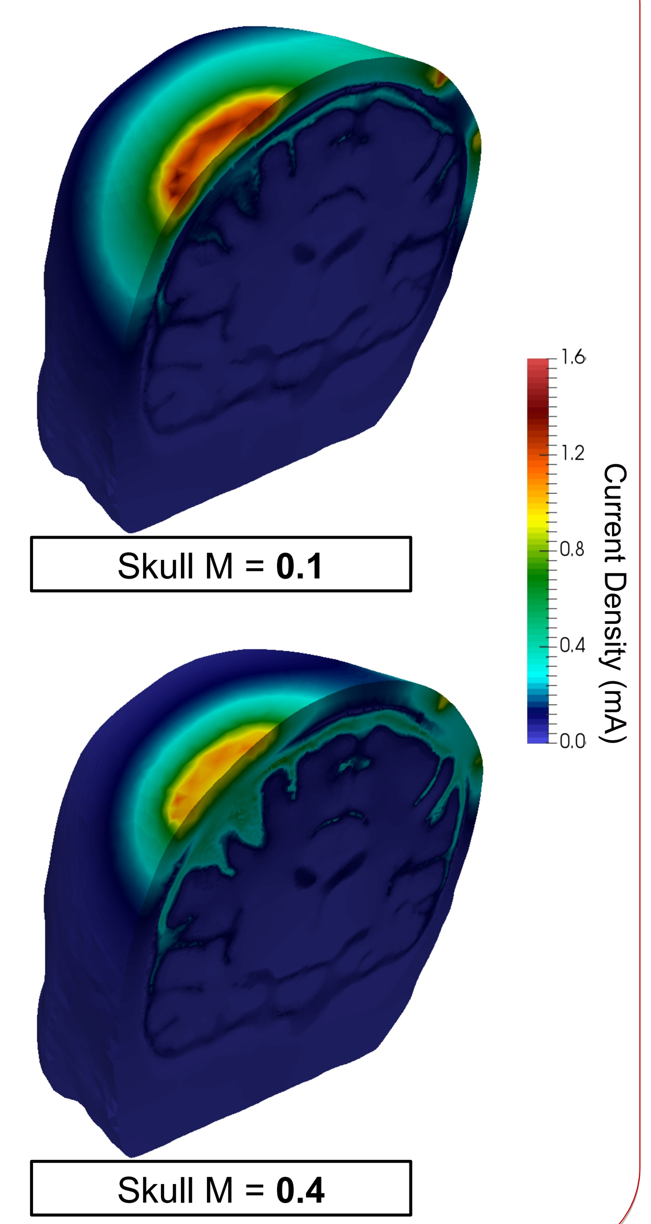
Current Progress

- Learning about numerical solution methods for PDEs and the Laplace Equation
- Learning to implement simulations to run TES numerical experiments
- Implementing stochastic simulator through a random number generator based on biological means and standard deviations



Next Steps

1. Large-scale simulations with variable skull conductivity values
2. Large-scale simulations, with variability in all brain tissue conductivity values
3. Migrate stochastic code to MRI-derived head geometry
4. Identify distinct, disease specific, electrode configurations for PDE system boundary conditions



References

[1] M. A. Nitsche, L. G. Cohen, E. M. Wassermann et al., "Transcranial direct current stimulation: State of the art 2008," Brain Stimulation, vol. 1, no. 3, pp. 206–223, 2008 [2] E. T. Dougherty, J. C. Turner, "An Object-Oriented Framework for Versatile Finite Element Based Simulations of Neurostimulation," Journal of Computational Medicine, vol. 2016, ID 9826596.

[3] E. T. Dougherty, J. C. Turner, and Frank Vogel, "Multiscale Coupling of Transcranial Direct Current Stimulation to Neuron Electrodynamics: Modeling the Influence of the Transcranial Electric Field on Neuronal Depolarization," Computational and Mathematical Methods in Medicine, vol. 2014, ID 360179, 2014